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MARKOVIAN MODELS IN RELIABILITY AND AVAILABILITY OF MARKOVIAN MODELS

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Abstract: **Assumption is made that the failure process and repair process whenever repairs are considered are Markovian. Champman Kolmogorov equations of Markov process are considered. System Reliability is obtained by Solving these equations. Availability of Markovian is discussed.**

Keywords: **Markovian Models.**

I. INTRODUCTION

Many authors have discussed a number of Systems under stable environment conditions (1,2) wholeline . If there is a repair facility, system may be repaired partially or completely to go back to operating state. Problem is solved by Laplace Transform. Expressions for MTTF are discussed.

II. ASSUMPTIONS

under failure of a component it is repaired and restored to be as good as new.

Formulations and Mathematical Analysis

Two component parallel system with a single repairman available for maintenance. Suppose failure and repair process are Markovian with rates a and b respectively.

Process $\{X(t);t\geq,0\}$ has 3 states i=0,1,2

The state i denoting number of failed units at the instanttine. The state 2 is an absorbing state. The process may be represented by a birth and death process having 3 states 0,1,2 state 2 is absorbing state.

Consider Kolmogoro v equations.

$p_i = P\{x(t)= i\}$

$p_0^1(t)$—2ap0(t)+bp1(t)

$p_1^1(t)=-(a+b)p_1(t)+2ap_0(t)$

$p^1_2\,(t)= a\,p_1(t)$ with initial conditions

$p_0(0)=-1$, $p_i\,(0)=0$ for i=1,2

With help of Laplace Transform and using initiitial conditions

$s\,(L\,p_0(t)-p_0(0)= -2a\,L\,(p_0(t) + bL\,(p_1\,(t)$

$s\,(p_1(t) - p_1(0) = -(a+b)\,L\,(p_1(t) + 2\,aL\,(p_0(t)$

$(s+2a)\,L\,(p_0(t)-b\,(Lp_1(t)=1$

& $(s+a+b)\,L(p_1(t)-2aL\,(p_0(t)=0$

Solving we get

$$L(p_1(t)= \frac{2a}{s^2+(3a+b)s+2a^2}$$

$$L\,(p_0(t)= \frac{a+b+s}{s^2+3(a+b)s+3a^2}$$

$L\,(p_1(t)$ can be written as

$$L\,(p_1(t) = \frac{2\,a}{[(s-s_1)\,(s-s_2)]} \qquad \text{where } si = \frac{-(3a+b) \pm \sqrt{(a+b)^2+4ab}}{2}$$

$$\text{Thus } p_1(t) = L^{-1}\,\frac{2a}{(s-s_1)\,(s-s_2)}$$

$$= \frac{2a}{s_1-s_2}\,L^{-1}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right] = \frac{2a}{s_1-s_2}\,[e^{s1t}-e^{s2t}]$$

$$\text{and } L\,(p_0(t) = \frac{a+b+s}{(s-s_1)\,(s-s_2)}$$

$$= \frac{1}{(s_1-s\,)}\,[(a=b+s1)e^{s_1t}] - (a+b+s_2)e^{s_2t}$$

System reliability is given by

$$R(t) = p_0(t) + p_1(t)$$

$$= \frac{(3a+b+s_1)\,e^{s_1t}-(3a+b+s_2)\,e^{s_2t}}{s_1-s_2}$$

Availiabity of a device is defined as probability that the device is operating Satisfactority at any time where total time includes operating time, active time repair time, logistic time etc.

The classification of availability are:

The point availability, Mean availability, Steady State availability, inherent availability and Operational availability

The point availability is $A\,(t)=R(t) + \int_0^t R(t-x)\;m(x)\,dx$

Where m(x) is renewal density function of the system.

If the failure time distribution and repair time distribution g(t) then point availability is given by

$$A(t) = L^{-1}\left[\frac{1-L\,w(t)}{s[1-L(w(t)\,L\,(g(t)]}\right]$$

Where w (t) is Commutative distribution function

Limiting availability is given by

$$A = \underset{t \to \infty}{Lt}\, A\,(t)$$

Limiting Reliability is $R\,(t) = \underset{t \to \infty}{Lt}\, 1-W(t)=0$

By definition

$$L\,(w(t)) = 1 - \frac{s}{\lambda}\int_0^{\infty} W\,(t)\,dt = 1$$

Where $1/\lambda$ is MTTF (Mean time to failure)

$$L\,(g(t) = 1 - \frac{s}{\mu}$$

Where $\frac{1}{\mu}$ is MTTR (Mean Time to repair)

$$A = \underset{t \to \infty}{Lt}\, A(t) = \underset{s \to 0}{LT}\, sL\,A(t)$$

$$A = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+\frac{1}{\mu}} = \frac{MTTF}{MTTF + MTTR}$$

Mean availability is proportion of time during a mission that a system is available for use. It represents the mean value of point availability function over period (O,T) and is given by

$$\overline{A(t)} = \frac{1}{t}\int_0^t A(u)\,du$$

Steady state availability for a single component, considering only corrective down time of system is given

$$A_1 = \frac{MTTF}{MTTF + MTTR}$$

For any system

$$A_1 = \frac{MTBF}{MTBF + MTTR}$$

Where MTBF is Mean time between failures.

Operational availability A_0 is the ratio of system uptime and total time. That is

$$A_0 = \frac{Uptime}{Operating\ cycle}$$

Where operating cycle is overall time period of operating being investigated and uptime is total time the system was functioning during operting circle

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